XIII. *On the oblique Refraction of Iceland Crystal. By* William Hyde Wollaston, *M. D. F. R. S.*

Read June 24, 1802.

In the preceding communication, I have inserted two different measures of refractive powers, distinctly observable in the Iceland crystal, as well as an estimate of its dispersive power; but have reserved for a separate treatise, some remarks which the same mode of investigation has enabled me to make on its oblique refraction.

The optical properties of this body have been so amply described by Huygens, in his *Traité de la Lumière*, that it could answer little purpose to attempt to make any addition to those which he has enumerated. But, as the law to which he has reduced the oblique refractions occasioned by it, could not be verified by former methods of measurement, without considerable difficulty, it may be worth while to offer a new and easy proof of the justness of his conclusions. For, since the theory by which he was guided in his inquiries, affords (as has lately been shown by Dr. Young*) a simple explanation of several phenomena not yet accounted for by any other hypothesis, it must be admitted that it is entitled to a higher degree of consideration than it has in general received.

According to that hypothesis, light proceeding from any luminous centre, is propagated by vibrations of a medium highly

* Bakerian Lecture. Phil. Trans. for 1801.

elastic, that pervades all space. In ordinary cases, the incipient undulations are of a spherical form; but, in the Iceland crystal, light appeared to HUYGENS to proceed as if the undulations were portions of an oblate spheroid, of which the axis is parallel to the short diagonal of an equilateral piece of the crystal, and its centre the point of incidence of the ray.

From this spheroidical form of the undulations, he deduces the obliquity of refraction; and lays down a law, observable in all refractions, at any surface of the spar, whether natural or artificial, which bears the closest analogy to that which obtains universally at other refracting surfaces; for as, in other cases, the ratio is given between the sine of incidence and sine of refraction, (or ordinate of the *spherical* undulation propagated,) so, in the Iceland crystal, the ratio between the sine of incidence and ordinate of refraction (in any one section of the *spheroidical* undulation) is a given ratio.

If ABD (Fig. 1, Plate XV.) be any surface of the spar, let FHOK be a section of the spheroid through its centre C, and RC any ray of light falling on that surface; draw FO a diameter of the spheroid, in the plane of incidence RVO, and CT, its semiconjugate diameter, in the plane of refraction FTO. Then, if CI be the refracted ray, VR, the sine of incidence, shall be to EI, the ordinate of refraction parallel to FC, in the constant ratio of a given line N to the semidiameter FC.

In any other plane of incidence, the ratio of sine to ordinate is also constant; but it is a different ratio, according to the magnitude of that diameter in which the plane of incidence intersects the ellipse FHOK.

When the incidence of a ray passing from any medium of greater density upon a surface of this spar, is such that the emergent ray becomes parallel to the surface, the ordinate of refraction is then a semidiameter of the spheroid; and, accordingly, the refractive power of this spar, when examined by means of a prism in different directions, should be found to vary as that semidiameter which coincides with the plane of incidence and refracting surface.

The observations that I have made on this substance, accord throughout with this hypothesis of HUYGENS; the measures that I have taken, correspond more nearly than could well happen to a false theory, and are the more to be depended on, as all my experiments, excepting the last, were made prior to my acquaintance with the theory, and their agreement was deduced by subsequent computation.

Exp. 1. The oblique refraction of this spar is rendered visible, by cementing a surface of it to a prism of flint-glass, with a little balsam of Tolu. When the line of sight bisects an acute angle of a natural surface of the spar, the refractive power is seen to be less than in any other direction, and may be expressed by the sine 1,488, or its reciprocal 0,67204.

Exp. 2. When the plane of incidence is parallel to one of the sides, the power is 1,518, of which the reciprocal is 0,6587.

Exp. 3. In a direction at right angles with either side, it is found still higher, being 1,537, or its reciprocal 0,6506.

Exp. 4. And, in the plane bisecting an obtuse angle, the refractive power of the natural surface appears greatest, and is expressed by the sine 1,571, or its reciprocal 0,6365.

Exp. 5. When either of the two greatest solid angles of the spar contained under three obtuse angles, is cut off by a polished surface making equal angles with each of its sides, the same refractive power 1,488 is found in all directions. By the theory also, the section of the spheroid is in this case a circle, and every semidiameter (FC) the same, since the plane is at right angles to the minor axis.

Exp. 6. If a plane surface be formed bisecting an obtuse angle of the spar, and applied to a prism, the same minimum of refraction 1,488, is found in a direction that coincides with the preceding plane, and therefore with the major axis of the generating ellipse; but, as the direction is varied, it increases so rapidly as soon to exceed the power of glass, and to be no longer ascertainable by the angle of incipient reflection.

Exp. 7. The regular refraction of this spar is also too great for examination by means of any prism, for want of a medium of union of sufficient density; but, by trial in the usual method, it measured, on an average of several experiments, 1,657, or its reciprocal 0,6035.

By assuming, as HUYGENS has done, the equality of this power with the maximum of the oblique refraction, we have sufficient data for construction of the spheroid by which the refractions are regulated; for we have 0,67204 (*Exp.* 1.) as major axis of the generating ellipse, and 0,6035 (*Exp.* 7.) will be the minor axis, parallel in position to the short axis of the spar.

The angle of inclination of this axis to the surfaces of the spar, if supposed to be equilateral, may be computed by spherical trigonometry, from any other angle that has been ascertained by measurement.

The measures that I have taken are not exactly those of HUYGENS; but I nevertheless hold them in equal estimation, from the conformity which I find they bear to each other, by assistance of his theory.

Exp. 8. I measured with care, an angle at which two surfaces of the spar are inclined to each other, and found it to be 105° 5′. Hence, the greater angle of the surfaces themselves may be computed to be 101° 55′; and the angle which the short axis makes with each plane surface is 45° 23′ 25″.

If GSMP (Fig. 2.) be a plane bisecting an obtuse angle of the spar, the section of the spheroid in that plane passes through the axis CS, and therefore is the generating ellipse. By calculating from the known dimensions of its major axis CP 0,67204, its minor axis CS 0,6035, and the angle GCS = 45° 23′ 25″, CG will be found* to be 0,6365, of which the reciprocal is 1,5736, differing but little from 1,571, as it appeared by measurement. (*Exp.* 4.)

Again, if ABDE (Fig. 4.) be one of the natural surfaces, and PG*p* the ellipse formed by that section of the spheroid, PC being as before 0,67204, and CG 0,6365, the reciprocal of 1,571 found by measurement, (*Exp.* 4.) then the semidiameter CT, parallel to the side AE, which makes an angle TCP 39° $2\frac{1}{2}$′, will be found to be 0,6573, instead of 0,6587, and its reciprocal 1,5215, instead of 1,518. (*Exp.* 2.)

The semidiameter also, in the direction of CL, perpendicular to the side, at an angle LCP 50° $57\frac{1}{2}$′, is found by calculation 0,650, and its reciprocal 1,539, instead of 0,6506 and 1,537. (*Exp.* 3.)

From the foregoing data, the course of a ray perpendicular to the surface of the spar may likewise be computed; for, since the sine of incidence is then nothing, the ordinate of refraction must be also nothing, and the ray will be refracted along the semiconjugate diameter CM. (Fig. 2.)

* (Fig. 3.) CS : CP :: tang. PCG : tang. PC*p*.
sec. PC*p* : sec. PCG :: CP : CG.

By calculation,* the angle which this conjugate makes with the perpendicular is 6° 7½′. But, by the following measurement, it appears to be 6° 16′.

Exp. 9. A piece of spar that measured 1,145 inch in thickness, was laid upon a line, and showed two images that were removed from each other $\frac{126}{1000}$ of an inch. Then, as 1,145 : 0,126 : : radius : tang. of 6° 16′.

The different results deduced from theory and from observation, will be seen at one view in the following statement.

In *Exp.* 2d,	observed	1,518 ;	calculated	1,5215.
3d,	———	1,537	———	1,539.
4th,	———	1,571	———	1,5736.
9th,	angle observed	6° 16′	- -	6° 7½′.

The angle observed differs from that obtained by computation, in a greater degree than any of the former measures; but, when the difficulty of measuring this angle with accuracy is considered, and also the greater effect of any incorrectness in the data from which a semiconjugate is computed, I think the result of this, as well of the preceding comparisons, must be admitted to be highly favourable to the HUYGENIAN theory; and, although the existence of two refractions at the same time, in the same substance, be not well accounted for, and still less their interchange with each other, when a ray of light is made to pass through a second piece of spar situated transversely to the first, yet the oblique refraction, when considered alone, seems nearly as well explained as any other optical phenomenon.

* (Fig. 5.) CS : CP :: tang. PCG : tang. *p*CO or co-tang. PCQ;
then CP : CS :: tang. PCQ : tang. PCM;
and LCP — PCM = MCL.

Fig. 1

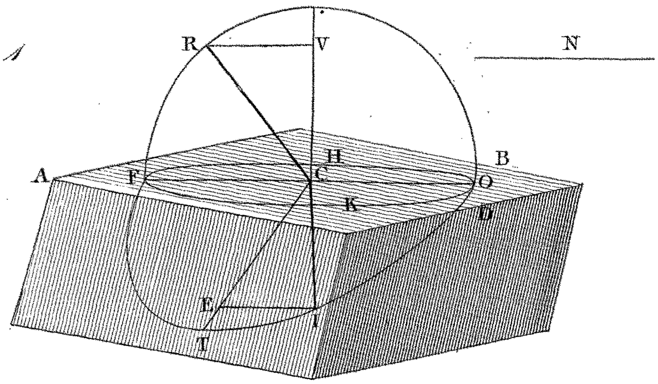

Fig. 2.

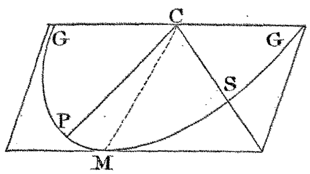

Fig. 3.

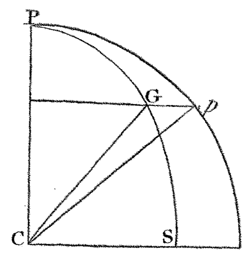

Fig. 4.

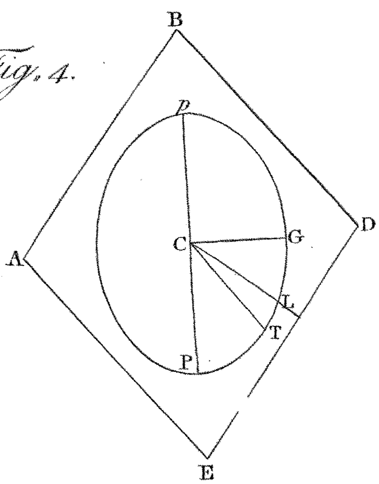

Fig. 5.